SYMPOSIUM ON "LINEAR MODELS AND DESIGN OF EXPERIMENTS"

Chairman : DR. K. KISHEN

Rapporteur : DR. A. DEY

The symposium was held during the 30th Annual Conference of the Indian Society of Agricultural Statistics on 31st December, 1976. The discussions were initiated by Dr. K. Kishen, the Chairman of the Symposium. Dr. Kishen made a brief survey of the contributions made in the recent past in the field of Linear Models and Design of Experiments with special reference to incomplete block designs, fractional factorials and Search linear models. After a brief introduction, he invited the speakers to present their papers. In what follows, brief summaries of the remarks made by the speakers are given.

1. *Linear Estimation in General Linear Models* by S.K. Mitra, Indian Statistical Institute :

Prof. Mitra discussed the problem of estimation of the parameters in a General Gauss Markov (GGM) model $(Y, X\beta, \sigma^2 V)$, *i.e.*, $Y=X\beta+e$, $E(e)=0$, $D(e)=\sigma^2 V$ where Y is a vector of order n of random variables, X a known matrix of order $n\times m$, β an unknown parameter vector of order m, σ^2 an unknown positive parameter and V, a non-negative definite (*n.n.d*) matrix of order n. To estimate the parameter vector β, several authors have considered various alternatives. Thus, Goldman and Zelen (1964) considered the minimisation of the quadratic form $(Y-X\beta)\ \bar{V}\ (Y-X\beta)$ where $\bar{V}$ is a generalised inverse of V. In order to investigate whether an analogue of the usual generalised least square (where V is positive definite) can be developed in the general case also, Rao and Mitra (1971) asked the following question :

Can we find a matrix M such that the estimate $\hat{\beta}$ of β is a stationary value of $(Y-X\beta)'\ M\ (Y-X\beta)$ and further an estimate of σ^2 obtained as $(Y-X\beta)'\ M\ (Y-X\beta) \div f$ where $f=R(V \vdots X)-R(X)$? It

has been shown by Rao and Mitra (1971) that one convenient choice of M is $M=(V+k^2 X X')^-$ for any choice of g-inverses, where k is an arbitrary constant, different from zero. Generalisations of this problem were considered by Mitra and Moore (1973, 1976) where V is of the form $V=\sigma_1^2 V_1+\ldots\ldots+\sigma_k^2 V_k$. Prof. Mitra illustrated the various procedures involved through a worked out example.

2. *Analysis of Designs with Two-way Elimination of Heterogeneity* by K.R. Shah, University of Waterloo, Canada and Indian Statistical Institute :

Dr. Shah's paper was concerned with the recovery of information from the row and column contrasts in a design eliminating heterogeneity in two directions. In this paper, Dr. Shah considered designs for which the treatment association matrices for the row design and for the column design commute. The class of such designs is very large and indeed all the designs given by Shrikhande (1951) belong to this class. Also, any design for which the row vs. treatment (or column vs. treatment) design is variance-balanced has this property.

A canonical form appropriate for these designs was developed. This canonical form was then utilized to show that for the procedures of combined estimation usually employed, the combined estimates of the treatment differences are unbiased.

Finally, a sub-class of designs was considered by Dr. Shah in which the product of the two treatment association matrices has all elements equal. For these designs, a procedure of combined estimation was developed which ensures uniform improvement over the so called interaction estimates. The speaker concluded by giving two illustrative examples of 2-way designs which have a factorial structure.

3. *Augmented Designs* by A.K. Nigam & P.N. Bhargava *I.A.R.S., New Delhi* :

This paper was presented by Shri P.N. Bhargava. Augmented designs are useful in many experimental situations where new strains or varieties are to be compared with certain established varieties in a single trial and sufficient material for the new varieties is not available.

In this paper, a method of constructing augmented designs for eliminating heterogeneity in one direction through orthogonal partitioning of a Latin square was discussed. The designs so obtained

belong to a class of Partially Efficiency-Balanced designs developed by Puri and Nigam (1976). A simple procedure for the analysis of these augmented designs was discussed.

4. *Linear Estimation in Designing Breeding Trials* by S.N. Sen, Govt. of Bihar, Patna :

Dr. Sen's paper related to the methods of linear estimation as applied to analysis of breeding data. In particular, Dr. Sen discussed how the genotypic variances be estimated from a variance components analysis in the case where all the progenies tested belong to different families. Dr. Sen also described how the repeatability and heritability estimates can be obtained from an appropriate variance components model.

5. *Irregular Tractions of 2^v Factorials from Doubly Balanced Incomplete Block Designs* by S. Mohanty, Orissa University of Agriculture & Technology, Bhubaneshwar.

In this paper, a special class of fractional plans for 2^v factorials was developed. These plans are obtained through a special series of doubly balanced incomplete block designs with parameters.

$v=4(t+1)$, $b=2(4t+3)$, $r=4t+3$, $k=2t+2$,
$\lambda=2\ t+1$ and $\delta=t$

where δ is the number of blocks in which any triplet of treatments appear together. These plans permit the estimation of all main-effects and a specified set of 2-factor interactions when three-factor and higher order interactions are assumed to be negligible. Dr. Monanty also briefly discussed the steps of analysis of this class of designs and showed that the variance of main-effect estimates obtained through this plan was smaller than the "index number" plans of Banerjee and some other resolution IV plans. Further, it was shown that in many situations, the proposed plans require a smaller member of runs than the existing plans.

6. *Incomplete Block Designs for Triallel Crosses* by K.N. Ponnuswamy, University of Madras :

A triallel cross is a set of all possible three-way crosses among a given set or parental lines. The utility of such crosses has been found in most commercial hybrids of corn, in pig breeding, silk worm breeding and in poultry breeding. Dr. Ponnuswamy's paper dealt with certain aspects of designing and analysis of triallel crosses.

In a triallel cross involving v parental lines, each parental line appears $(v-1)(v-2)$ times among the crosses as a grand parent and in $(v-1)(v-2)/2$ crosses as a parent. Due to this fact, there is a type of imbalance in the precision of the estimates of the half-parental and parental effects. Dr. Ponnuswamy described an incomplete block design which corrects this imbalance. The design is constructed simply by taking the constituents of the i-th block all those crosses which have the i-th line as a grand parent. It was shown that this type of design is a *PBIB* design with three associate classes involving two replications. Dr. Ponnuswamy then discussed the procedure of analysis of these triallel cross design and demonstrated how the genetic parameters of interest can be estimated through the design.

After the speakers had presented their respective papers, the Chairman requested Prof. J.N. Srivastava of the Colorado State University to sum up the discussion. Dr. Srivastava thanked all the speakers for their contributions and made suggestions for improvement in respect of some of the papers. In particular he remarked that the commuting property of the treatment association matrices used by Dr. Shah in his paper is a very useful one and has other applications as well. Regarding the paper on Augmented Designs by P.N. Bhargava, Dr. Srivastava felt that this was a very good field of research and studies in respect of optimal augmented designs should provide a worthwhile area of research. Regarding the fractional factorial plans of Dr. Mohanty, Dr. Srivastava observed that Doubly balanced designs are closely related to balanced arrays of strength three and these arrays are known to provide fractional factorial plans of resolution IV. Though the series of designs presented by Dr. Mohanty is interesting, Dr. Srivastava felt that a deeper study into this problem would be welcome.

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